# Oscillatory behavior of a superradiating system coupled to electron reservoirs

T. Brandes*, J. Inoue, and A. Shimizu
*Institute of Physics, University of Tokyo, 3–8–1 Komaba, Tokyo 153–8902, Japan*
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We investigate a superradiating system coupled to external reservoirs. Under conditions where electrons tunneling at a rate $T$ act like an electron pump, we predict a novel phenomenon in the form of oscillations with a frequency $\omega \simeq \sqrt{2\Gamma T}$ that appear in the (photon) emission intensity, where $\Gamma$ is the spontaneous decay rate of a single two-level system. The effect, together with a strong enhancement of the superradiant peak, should be observable in semiconductor quantum wells in strong magnetic fields, or in quantum dot arrays.

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Superradiance occurs in the spontaneous coherent decay of an initially excited ensemble of $N$ two-level systems which are interacting with a common photon field. The corresponding emission rate of photons is proportional to $N^2$ which is abnormally large when compared to the incoherent decay of $N$ independent systems. Furthermore, the emission is not exponentially in time but has the form of a very sudden peak on a short time scale $\sim 1/N$. The phenomenon was predicted by Dicke in a seminal paper [1] and observed for the first time [2] in an optically pumped hydrogen fluoride gas. Since then, many investigations of superradiance concentrated on modifications through geometry effects and dephasing processes such as dipole-dipole interactions which had been neglected in the original Dicke paper [3]. Since superradiance intrinsically is a many-body problem, this also gave the possibility to study the concept of coherence and dephasing in a many-body context [4].

In the present Letter, we propose an extension of the original Dicke model that in principle opens the possibility for an external control of superradiance. The main idea is to realize the superradiant 'active region' in a semiconductor and to couple it to external electron reservoirs through tunnel barriers, thus allowing for a varying electron number. We predict that for reservoir conditions which work like an electron pump, the initial coherent superradiant peak can be strongly enhanced if the tunnel rate $T$ is high enough. Furthermore, we predict a novel phenomenon in the form of strong oscillations of the emitted light with a frequency $\omega$ that in good approximation is given by

$$\omega \simeq \sqrt{2\Gamma T} \tag{1}$$

for large $T > 2\Gamma$, where $\Gamma$ is the spontaneous decay rate of a single two-level system. For smaller tunnel rates $T$, there is a smooth crossover to the conventional Dicke peak [1] in the limit $T \to 0$. In contrast to oscillatory superradiance in atomic systems [5], these oscillations are not due to reabsorption of photons, but due to tunneling of electrons into an active region which is described by many-body wave functions. The latter are characterized by a total pseudo spin $J$ and a pseudo spin projection $M$ for $N$ electrons occupying upper and/or lower levels, including empty levels. In contrast to the original Dicke problem, $J$ is no longer conserved but develops a dynamics that is driven by the tunneling process which leads to a coupling of $J$ and $M$, whose equations of motion have oscillatory solutions.

We propose a concrete realization of the model in a semiconductor quantum well in a strong magnetic field which defines degenerate energy levels for electrons and holes. The 'active layer' is driven in analogy to a semiconductor diode laser by injecting conduction band electrons and valence band holes that can radiatively recombine. In contrast to the diode laser, no stimulated emission processes are required.

We first describe our model Hamiltonian. A photon field $H_p = \sum_{\mathbf{Q}} \Omega_{\mathbf{Q}} a^\dagger_{\mathbf{Q}} a_{\mathbf{Q}}$ with creation operator $a^\dagger_{\mathbf{Q}}$ for a mode $\mathbf{Q}$ gives rise to transitions which change an internal degree of freedom $\sigma = (\uparrow, \downarrow)$ of one-particle electronic states labeled $(i, \sigma)$ with creation operator $c^\dagger_{i,\sigma}$. The states are degenerate with respect to $i$ with energies $\varepsilon_{i\uparrow} = -\varepsilon_{i\downarrow} = \hbar\omega_0/2$. The electron–photon coupling matrix element $g_{\mathbf{Q}}$ is assumed to be independent of the electronic quantum numbers $(i, \sigma)$. This condition is fulfilled if the momentum matrix element $\langle i\sigma|\mathbf{p}|i'\sigma'\rangle \propto \delta_{ii'}$ for $\sigma \neq \sigma'$ as is the case for the applications discussed below. Then, the Hamiltonian without reservoir coupling can be written as

$$H_D = \hbar\omega_0 \hat{J}_z + \sum_{\mathbf{Q}} g_{\mathbf{Q}} \left( a^\dagger_{\mathbf{Q}} + a_{\mathbf{Q}} \right) \left( \hat{J}_+ + \hat{J}_- \right) + H_p, \tag{2}$$

where the operators $\hat{J}_+ := \sum_i c^\dagger_{i,\uparrow} c_{i,\downarrow}$, $\hat{J}_- := \sum_i c^\dagger_{i,\downarrow} c_{i,\uparrow}$ and $\hat{J}_z := \frac{1}{2} \sum_i \left( c^\dagger_{i,\uparrow} c_{i,\uparrow} - c^\dagger_{i,\downarrow} c_{i,\downarrow} \right)$ form a (pseudo) spin algebra with angular momentum commutation relations. Note that so far we have not refered to the explicit form of the one-particle states; the Hamiltonian Eq. (2) represents a whole class of physical systems rather than one specific experimental situation. The model and the examples below refer to a photon field; we point out, however, that our results are valid for a coupling to an arbitrary boson field (e.g. phonons), too.

We allow the number of electrons $N$ in the active region as described by $H_D$ to vary by tunneling to and from electron reservoirs $\alpha = I/O$ ('In'and 'Out') with Hamiltonians $H_\alpha = \sum_k \varepsilon_k^\alpha c_{k,\alpha}^\dagger c_{k,\alpha}$ for non-interacting electrons with equilibrium Fermi distributions $f_\alpha$. The tunneling of electrons is described by the usual tunnel Hamiltonian $H_T = \sum_{ki\sigma\alpha} \left( t_{ki\sigma}^\alpha c_{k,\alpha}^\dagger c_{i,\sigma} + c.c. \right)$ with coefficients $t_{ki\sigma}^\alpha$ that in general depend on the specific form of the tunnel barriers in real space and the one-particle wave functions. The total Hamiltonian is given by the sum $H = H_D + \sum_\alpha H_\alpha + H_T$. We note that in this Letter we do not consider explicitly the effects of electron-electron and electron-hole interactions but concentrate on the effects which result from the tunneling processes only. We do not expect the interactions to lead to qualitative differences, because in our case the carrier density is high and drastically varies as a function of time, which weakens and smears out the interaction effects.

The coupling of the active region to reservoirs combines the dynamics of a (many) spin-boson problem, $H_D$, with the transport through it. We use the master equation approach to calculate the dynamical evolution of observables like the emission intensity. This method, while only perturbative in the coupling matrix elements $g_{\mathbf{Q}}$ and $t^\alpha$, has turned out to yield results that seem to be in good agreement with experiments both for the superradiant problem [3,5] and transport through small regions of interacting electrons (quantum dots) [6,7]. We note that correlations between different tunneling processes (co-tunneling) are neglected here.

The eigenstates of the active region are characterized by the total pseudo spin $J$ and its projection $M$ through $\hat{J}^2 |JM;\{\lambda\}\rangle = J(J+1)|JM;\{\lambda\}\rangle$ and $\hat{J}_z|JM;\{\lambda\}\rangle = M|JM;\{\lambda\}\rangle$, where $\hat{J}$ is the total pseudo spin operator. Here, $\{\lambda\}$ denotes all additional quantum numbers apart from $J$ and $M$ that are necessary to characterize the eigenstates of $H_D$. Radiative transitions obey the selection rule $M \to M \pm 1$ with a spontaneous emission intensity $I_{JM}$,

$$I_{JM} = \hbar\omega_0 \Gamma \nu_{JM}, \quad \nu_{JM} := (J+M)(J-M+1). \quad (3)$$

Here, $\Gamma$ is the spontaneous emission rate of one *single* two-level system; for radiative transition in atoms, $1/\Gamma$ is in the nano second range.

Furthermore, the rates $\Gamma_{JM\to J'M'}$ for transitions between eigenstates of $H_D$ through electron tunneling may be written in the form

$$\Gamma_{\ JM\to J'M'} = \sum_\alpha T^\alpha \left\{ \gamma_{JM\to J'M'} f_\alpha(\Delta E) + \gamma_{J'M'\to JM} \left(1 - f_\alpha(-\Delta E)\right) \right\}. \quad (4)$$

Here, $T^\alpha := (2\pi/\hbar) \sum_k t_{ki\sigma}^\alpha (t_{ki'\sigma'}^\alpha)^* \delta(\Delta E - \varepsilon_k^\alpha)$ is the tunnel rate for lead $\alpha$ which in fact is a matrix in $(i,\sigma)$ and depends on the energy difference $\Delta E$ between final and initial state. We neglect the energy and site dependence, furthermore the dependence on $\sigma$ is absorbed into the index $\alpha$ through the boundary conditions: the coupling to the reservoirs has the effect of a 'pseudo spin-up pump' where only $\sigma = \uparrow$ electrons can tunnel in and $\sigma = \downarrow$ electrons tunnel out. That is, the chemical potential for $\alpha = I$ is assumed to be situated above all possible energy differences of states $|JM;\{\lambda\}\rangle$ with $M$ differing by plus $1/2$, and the chemical potential for $\alpha = O$ is chosen such that 'down' electrons can tunnel out to $\alpha = O$, but not tunnel in. Furthermore, tunnel matrix elements for processes like tunneling from $\alpha = I$ with $\sigma = \downarrow$, leading to $M' = M - 1/2$, are assumed vanishingly small; this condition is satisfied in the applications below.

The coefficients $\gamma$ in Eq. (4) are determined by the Clebsch-Gordan coefficients for adding or removing a single pseudo-up or down spin to the active region, $\gamma_{JM\to J'M'} := |\sum_{m=\pm 1/2} \langle J'M'|JM; j=1/2, m\rangle|^2$. Here, we adopted the same approximation that has been used in [6] for calculating matrix elements for transition rates in transport through correlated quantum dots: the specific form of the many-particle wave function in the active region (i.e. the quantum numbers $\{\lambda\}$) is neglected, and the matrix elements are approximated by the Clebsch-Gordan coefficients.

We describe the dynamics of the active region in terms of probabilities $\rho(JM)_t$ which are the diagonal elements of the reduced density operator at time $t$ in the basis of eigenstates $|JM\{\lambda\}\rangle$, where all information apart from the quantum numbers $J$ and $M$ has been traced out. In lowest order perturbation theory, there is no interference between $H_{ep}$ and $H_T$, and the master equation in Markov and Born approximation reads [9]

$$\dot{\rho}\ (JM)_t = -\Gamma \left\{ \nu_{JM} \rho(JM)_t - \nu_{JM+1} \rho(JM+1)_t \right\} + \sum_{J'M'} \left\{ \Gamma_{J'M'\to JM} \rho(J'M')_t - \Gamma_{JM\to J'M'} \rho(JM)_t \right\}. \quad (5)$$

The time dependence of the expectation value of the emission rate, Eq. (3), was obtained from the time evolution of $\rho(JM)_t$ by numerical solution of Eq. (5). The result is shown in Fig. (1) for an initially excited state with $J=6$, $M=5$. The initial emission maximum is the original 'Dicke-peak', followed by oscillations that die out at an intensity proportional to the tunnel rate $T$ that was chosen symmetric, $T = T^I = T^O$. The frequency of the oscillations increases with $\sqrt{T}$ and follows in good approximation the law Eq. (1). In particular, the initial peak is strongly enhanced with increasing tunnel rate. This behavior is related to an initial increase of the total pseudo spin as can be seen from the 'phase-space' plot (inset of Fig. (1)) of the expectation values of $J$ and $M$ which both oscillate in time.

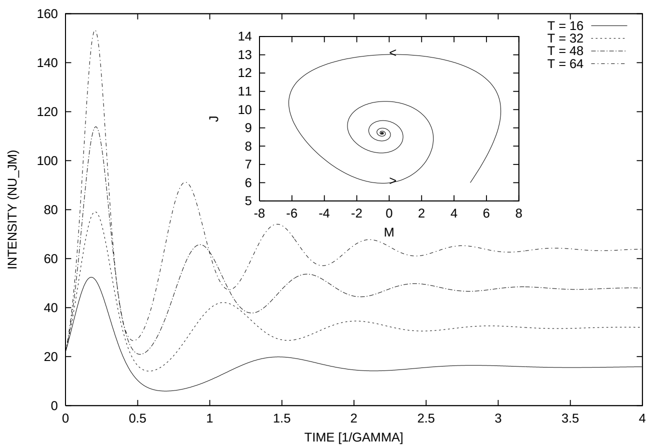


FIG. 1. Time evolution of the emission intensity $\nu_{JM} = I_{JM}/\Gamma\hbar\omega_0$ for different transmission rates $T$. Inset: $\langle J\rangle_t$ vs. $\langle M\rangle_t$ for $T = 64$.

These observations can be understood as follows. In the original Dicke problem, a fixed total number $N$ of electrons with exactly one pseudo spin $\sigma$ on each site $i$ is assumed. Here, we work in a grand-canonical ensemble where $N$ varies through single electron tunneling: doubly occupied or empty single particle levels $i$ become possible, i.e. there is no longer an exact 'half-filling'. One immediate consequence is that a tunneling electron changes the quantum numbers $J$ and $M$. The change $\dot{J}$ of $J$ is proportional to $M$ itself, $\dot{J}(t) = TM(t)/J(t)$, which follows considering the Clebsch-Gordan coefficients for adding a pseudo up spin. At the same time, $M$ is increased by 1/2 as is the case for out–tunneling of a pseudo down spin: $M$ increases by 1 at the tunnel rate $T$ and decreases by spontaneous emission at a rate $\Gamma\nu_{JM}$. Therefore, $J$ and $M$ obey roughly

$$\begin{aligned} \dot{M}(t) &= -\Gamma\nu_{J(t)M(t)} + T \\ \dot{J}(t) &= T \cdot M(t)/J(t), \end{aligned} \tag{6}$$

which are governed by the two parameters $\Gamma$ and $T$, the emission rate and the tunnel rate. After eliminating $J$ in the equations of motion Eq. (6), and approximating $\nu_{JM} \approx J^2 - M^2$, we obtain $\ddot{M} - 2\Gamma M\dot{M} + \omega^2 M = 0$ with $\omega = \sqrt{2\Gamma T}$, Eq. (1). For $T > 2\Gamma$, this describes a harmonic oscillator with frequency Eq. (1) and amplitude dependent damping. For smaller $T$, the oscillations are no longer visible and Eq. (1) does no longer hold. For $T \to 0$, there is a smooth crossover to the conventional Dicke peak with vanishing intensity at large times and without oscillations. We again point out that our model applies to the small sample limit of the superradiant problem: reabsorption processes of photons that may lead to oscillatory behavior of the intensity do not play any role here.

The oscillator equations Eq. (6) in fact are the quasi-classical limit of the master equation Eq. (5) for $J \gg 1$. Neglecting fluctuations of the expectations values $M(t)$ and $J(t)$, one writes [9] the time dependent probability distribution as $\rho(JM)_t = \delta_{M,M(t)}\delta_{J,J(t)}$. The detailed form of the intensity peak and the intensity oscillations obtained in this way deviate from the exact solution of Eq. (5), whereas the qualitative features remain unchanged which we checked numerically. In the limit of large $J$, one obtains the two equations Eq. (6) for $J$ and $M$.

We now turn to the question in what physical systems the effects described above can be observed experimentally. We note that the tunneling processes can be replaced with classical injection processes over potential barriers, because we have assumed that quantum correlation is absent between subsequent tunneling processes. We thus propose the system of electrons and holes in semiconductor quantum wells in strong magnetic fields. Vertical injection of conduction-band electrons and valence-band holes into an active region acts like the pumping mechanism described above. In fact, this mechanism is exactly what is used in lasers or light emitting diodes with forward biased pn junctions. In our case, mirrors as in a laser are not necessary, in particular stimulated emission processes must play no role. The strong magnetic field is necessary to have dispersionless single electron levels $i = X$, corresponding to the lowest Landau bands ($n = 0$) and guiding center $X$ [8] in the conduction and the valence bands. In this case, the interband optical matrix elements are diagonal in $i$. The correspondence with our model can be seen by mapping its four basic single particle states to the states of the electron-hole system (Fig. (2)): the empty state becomes the hole (H), the pseudo-spin down electron becomes the empty state, the doubly occupied state becomes the electron (E), and the pseudo-spin up electron becomes the state with one electron and one hole. The number of total electrons $N$ in our model has its correspondence via

$$N = N_E + N_s - N_H, \tag{7}$$

where $N_s = \Phi/\Phi_0$ is the degeneracy for a given magnetic flux $\Phi$ ($\Phi_0 = hc/e$ is the flux quantum), $N_E$ the number of electrons in the conduction-band, and $N_H$ the number of holes in the valence-band.

We predict that an initial optical or current excitation of the system leads to a superradiant peak of emitted light that becomes strongly enhanced if the tunneling rate becomes higher. Furthermore, subsequent oscillations of the emitted light should be visible at an approximate frequency Eq. (1). We also expect similar oscillations to be visible as weak corrections to the injection current; detailed calculations will be presented elsewhere.

As a second experimental setup, we propose an array of identical quantum dots, coupled to electron reservoirs as above. The array must have the capability to coherently radiate, where each dot has a pair of well-defined internal levels that allow for transitions under emission

of photons. Another possible realization could be a geometry as in the quantum cascade laser which has been proposed recently [10] as an alternative to conventional semiconductor diode lasers. In this case, transitions between different electronic subbands lead to photon emission.

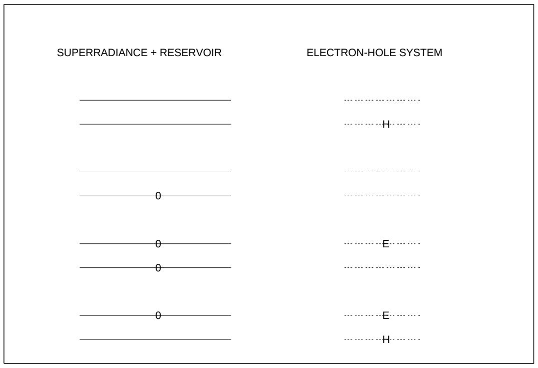


FIG. 2. Correspondence of our model (superradiance with reservoir) with an electron-hole system. 'E' denotes an electron in the conduction band, 'H' a hole in the valence band.

Note that in all three cases, the photon escape time $\tau = L/c$ ($L$ is the linear dimension of the active region and $c$ the speed of light) has to be much smaller than all other time scales of the problem, because in our model we assumed the 'small sample superradiance case' where reabsorption effects play no role.

We mention that all the effects described above should in principle be observable not only for photons, but also for other bosonic fields such as phonons, or magnons.

Finally, we point out that so far we have not addressed the question of phase coherence. In fact, the conventional superradiance is a transient process that occurs only on a 'mesocopic' time scale with an upper boundary [11] given by a phase coherence time $\tau_\phi$. Inelastic processes such as dipole-dipole interactions [4] in general destroy the phase coherence between single particle states and the description using the Dicke states with well-defined $J$ and $M$ becomes void. On the other hand, coherence between states with different $J$ and different $M$ is not required in our formalism, on the contrary this would require consideration of coherent tunneling which is beyond the scope of our approach. We assume that dephasing processes are weak such that the time to observe the initial Dicke peak and some cycles of the subsequent oscillations is still shorter than $\tau_\phi$. An ideal case would be a strong initial excitation to a high initial pseudo spin $J$, and a high tunnel rate $T$ that yields fast oscillations. Furthermore, strong magnetic fields in general suppress scattering rates although at the present state we can give no quantitative estimates for $\tau_\phi$.

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* New address: 1. Inst. Theor. Physik, Jungiusstr. 9, D-20355 Hamburg, Germany.


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